Low energy nuclear scattering and sub-threshold spectra from a multi-channel algebraic scattering theory.

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Abstract. A multi-channel algebraic scattering theory, to find solutions of coupled-channel scattering problems with interactions determined by collective models, has been structured to ensure that the Pauli principle is not violated. Positive (scattering) and negative (sub-threshold) solutions can be found to predict both the compound nucleus sub-threshold spectrum and all resonances due to coupled channel effects that occur on a smooth energy varying background.

1. INTRODUCTION

Low energy cross sections from the collision of nucleons with light mass nuclei show sharp as well as broad resonances upon a smooth, energy dependent background. Those resonances may correlate to states in the discrete spectrum of the target. To interpret such scattering data requires use of a complex coupled-channel reaction theory. We have developed such a theory [1]; one that has very important improvements over those used heretofore. This theory, a multi-channel algebraic scattering (MCAS) theory, finds solution of the coupled Lippmann-Schwinger equations for the scattering of quantal systems in momentum space, and is a quite general one. To date, however, we have limited study to nucleon scattering from targets of zero ground state spin.

The prime information sought are scattering (S) matrices which are easily extracted from the T-matrices generated by MCAS. The approach involves using matrix algebra on matrices built using Sturmian-state expansions of the relevant nucleon-nucleus potential matrix. With this method, all resonances in any energy range, can be identified and their centroids, widths, and spin-parities determined. Similarly the energies and spin-parities of bound states of the compound system sub-threshold can be determined.

It has long been known that collective model prescriptions of nucleon-nucleus scattering violate the Pauli principle. It seems that this has been the case in all such calculations made prior to those reported in Ref. [1]. However, the MCAS procedure enables use of an orthogonalizing pseudo-potential (OPP) approximation by which such Pauli principle violation can be alleviated. Doing so is crucial to finding the parameter values specifying

the interaction that simultaneously gives the sub-threshold compound nucleus spectrum and the low energy scattering cross sections.

The MCAS approach that is based upon using Sturmian functions as a basis set, is outlined next along with the process by which resonances can be identified and located. Results of calculations made using a collective model prescription for the interaction potential matrix are then discussed. In that collective model, the interaction field is allowed to be deformed from sphericity. That deformation has been taken to second order in the case of neutron scattering from ^{12}C allowing coupling to the ground 0^+_1, 2^+_1 (4.4389 MeV), and 0^+_2 (7.64 MeV) states. Results have been found for nucleon energies to 6 MeV. Some new results for the ^{6}He+p system are reported as well.

The theory also allows formation of the optical potential by appropriately subsuming the coupled channels equations into effective elastic channel scattering equations. The optical potentials thus constructed, as well as allowing for the Pauli principle, are very non-local and energy dependent. We present typical results for the $n+^{12}$C case.

2. THE MCAS THEORY (IN BRIEF)

For a system of Γ channels for each allowed scattering spin-parity J^π let the index $c\ (=1,\Gamma)$ denote the set of quantum numbers that identify each channel uniquely. Let $c=1$ designate the elastic channel. The integral equation approach in momentum space for potential matrices $V^{J^\pi}_{cc'}(p,q)$, requires solution of coupled Lippmann-Schwinger (LS) equations giving a multichannel T-matrix of the form

$$\begin{aligned} T_{cc'}(p,q;E) &= V_{cc'}(p,q)+\frac{2\mu}{\hbar^2}\left[\sum_{c''=1}^{\text{open}}\int_0^\infty V_{cc''}(p,x)\frac{x^2}{k^2_{c''}-x^2+i\varepsilon}T_{c''c'}(x,q;E)\,dx\right. \\ &\left.-\sum_{c''=1}^{\text{closed}}\int_0^\infty V_{cc''}(p,x)\frac{x^2}{h^2_{c''}+x^2}T_{c''c'}(x,q;E)\,dx\right], \quad (1) \end{aligned}$$

where the conserved values of J^π is to be understood. The open and closed channels contributions have channel wave numbers $k_c=\sqrt{\frac{2\mu}{\hbar^2}(E-\varepsilon_c)}$ and $h_c=\sqrt{\frac{2\mu}{\hbar^2}(\varepsilon_c-E)}$, for $E>\varepsilon_c$ and $E<\varepsilon_c$ respectively. ε_c is the threshold energy of channel c and μ is the reduced mass. Solutions of Eq. (1) are sought using (finite rank) separable expansions of the potential matrix elements, $V_{cc'}(p,q)\sim\sum_{n=1}^N \chi_{cn}(p)\ \eta_n^{-1}\ \chi_{c'n}(q)$.

In the MCAS method, the Sturmians that lead to specification of the form factors $\chi_{cn}(p)$, are solutions of Schrödinger equations with the chosen input matrix of potentials. In coordinate space, if those potentials are defined from a collective model prescription of the nucleon-target system with local forms $V_{cc'}(r)$, the Pauli principle can be satisfied by using an OPP method in the determination of the Sturmians [1, 2].

The link between the multichannel T- and S-matrices is

$$S_{cc'}=\delta_{cc'}-i^{l_{c'}-l_c+1}\pi\frac{2\mu}{\hbar^2}\sum_{n,n'=1}^N\sqrt{k_c}\chi_{cn}(k_c)\left([\eta-\mathbf{G}_0]^{-1}\right)_{nn'}\ \chi_{c'n'}(k_{c'})\sqrt{k_{c'}}\ , \quad (2)$$

where now c, c' refer to open channels only. In this representation, η has matrix elements $(\eta)_{nn'} = \eta_n\ \delta_{nn'}$ while those of $\mathbf{G}_0$ are $(G_0)_{nn'} = (G)$,

$$(G) = \frac{2\mu}{\hbar^2}\left[\sum_{c=1}^{\text{open}} \int_0^\infty \chi_{cn}(x)\frac{x^2}{k_c^2 - x^2 + i\varepsilon}\chi_{cn'}(x)\ dx - \sum_{c=1}^{\text{closed}} \int_0^\infty \chi_{cn}(x)\frac{x^2}{h_c^2 + x^2}\chi_{cn'}(x)\ dx\right], \tag{3}$$

Bound states of the compound system are defined by the zeros of the matrix determinant when the energy is $E < 0$. They link to zeros of $\{|\eta - \mathbf{G}_0|\}$ when all channels in Eq. (3) are closed. For details of the Sturmian expansions and their use, see Ref. [1].

An essential feature of the MCAS approach, of importance in studies when very narrow resonances are to be observed, is the resonance finding scheme [1]. Essentially that requires recasting the elastic scattering S-matrix (for each J^π) as

$$S_{11} = 1 - i\pi\frac{2\mu}{\hbar^2}\sum_{nn'=1}^{M} k\ \chi_{1n}(k)\frac{1}{\sqrt{\eta_n}}\left[\left(\mathbf{1} - \eta^{-\frac{1}{2}}\mathbf{G}_0\eta^{-\frac{1}{2}}\right)^{-1}\right]_{nn'}\frac{1}{\sqrt{\eta_{n'}}}\chi_{1n'}(k)\,. \tag{4}$$

Here, the elements of the diagonal (complex) matrix $\eta^{-\frac{1}{2}}$ are $\frac{1}{\sqrt{\eta_n}}\delta_{nn'}$. Then the complex-symmetric matrix, $\eta^{-\frac{1}{2}}\mathbf{G}_0\ \eta^{-\frac{1}{2}}$ can be diagonalized to find the evolution of its complex eigenvalues, ζ_r, with respect to energy. Resonant behavior occurs when one of the complex ζ_r eigenvalues passes close to the point (1,0) in the Gauss plane. The complex part of that limit eigenvalue relates to the width of the resonance. The elastic channel S-matrix has a pole structure at the corresponding energy where one of these eigenvalues approach unity.

3. RESULTS USING MCAS FOR $N + A$ SYSTEMS

Results for the scattering of neutrons to ∼6 MeV from ^{12}C are displayed in Fig. 1. Therein the elastic scattering cross section is compared with data with respect to the ground state of ^{12}C as zero energy. On the same scale, in the right hand parts of the figure, the experimental and theoretical sub-threshold and resonance states in ^{13}C are compared. We have noted in a recent publication [2] how crucial it is to account for the Pauli principle via the OPP scheme to achieve these results.

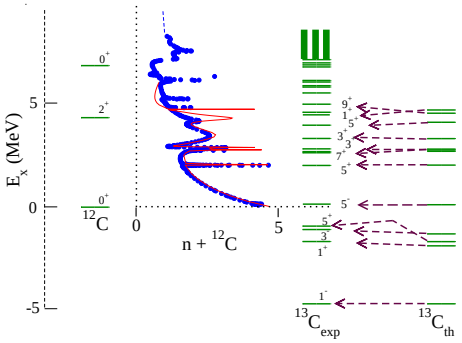
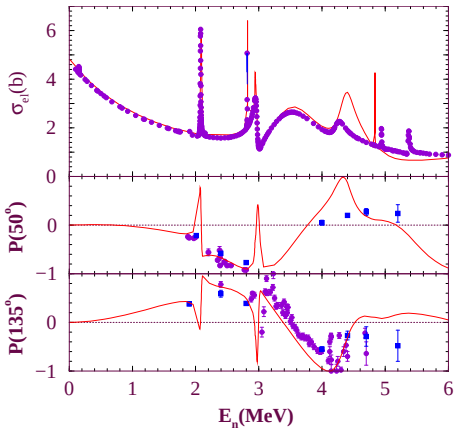


FIGURE 1. Spectra of ^{12,13}C and the elastic cross section (barn) for the $n + ^{12}$C system. The labels with the states designate values of $2J$ and parity.

In Fig. 2., the elastic cross section for neutron scattering is shown in the top panel



while polarizations at two scattering angles are shown underneath. The resonance structures most evident in the cross section are well reproduced by MCAS and those details are confirmed in the polarizations. The spin-parities of the resonances also coincide with those of known states in the spectrum of ^{13}N. Notably the

FIGURE 2. The elastic cross section for n-^{12}C scattering (top) and polarizations at 50° (middle) and at 135° (bottom) compared to MCAS results.

prominent $\frac{5}{2}^+$ resonance at 2 MeV, and the two $\frac{3}{2}^+$ resonances spanning 2.8 - 4.0 MeV, are found with very good widths, narrow and broad respectively. In Fig. 3., we show the spectra that one obtains for the $p+{}^6$He system when the target (^{6}He) structure is taken as three states; the 0^+ (ground), and two 2^+ states at 1.78 and 5.68 MeV respectively. A quadrupole deformation was considered and first and second order deformations allowed for each excitation.

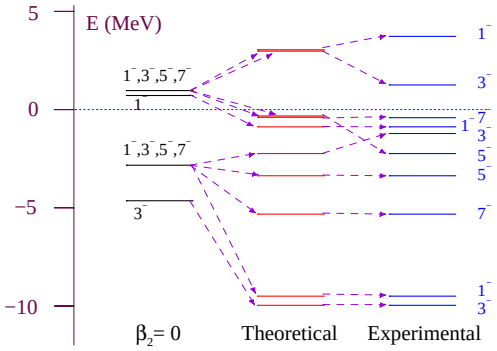


FIGURE 3. The spectra of ^{7}Li as determined using MCAS for the $p+{}^6$He system.

The base interaction potential required a large diffuseness consistent with the target having an extended neutron matter distribution. The results on the far left came from evaluations with deformation set to zero and show the sub-structural origins of each state in the actual spectrum [3]. Again use of the OPP to account for Pauli blocking of occupied states was crucial in finding these results.

4. MCAS AND THE OPTICAL POTENTIAL

Assuming a local form for the elastic channel element of the potential matrix, the optical potential for elastic scattering is defined by

$$V^{opt}(r,r';E) = V_{11}(r) + \sum_{c,c'=2}^{\Gamma} V_{1c}(r)\, G^{(Q)}_{cc'}(r,r';E)\, V_{c'1}(r')\ , \qquad (5)$$

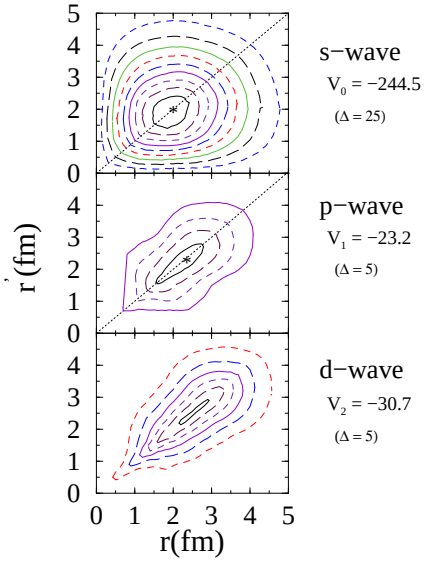
where the second term is the dynamic polarization potential (DPP) $\Delta U(r,r';E)$. This makes the formulated optical potential complex (if the energy allows more than one

open channel), nonlocal, and energy dependent as $G^{(Q)}_{cc'}$ are the full Green functions referring to the $Q\ (=\Gamma-1)$ excluded channels.

In the MCAS approach [1], with $\Lambda(E)=\left[\eta-\mathbf{G}^{(Q)}_0(E)\right]^{-1}-\eta^{-1}$, the DPP is

$$\Delta U(r,r';E)=\sum_{n,n'=1}^{N}\chi_{1n}(r)\left[\Lambda(E)\right]_{nn'}(E)\chi_{n'1}(r')\ . \tag{6}$$

where $\chi_{n'1}(r)$ are Bessel transforms of the from factors $\chi_{n'1}(k)$.



The energy-volumes of the DPP evaluated for 2.73 MeV neutrons on ^{12}C are plotted in Fig. 4. for three partial waves, $\ell=0,1,2$. The central depths of each are indicated on the right as are the energy steps of the contour lines. As the energy is below the first inelastic threshold, the DPP is purely real. But the coupled channel effects make that aspect of the optical potential extremely non-local. The DPP is also very angular momentum dependent.

FIGURE 4. The DPP for three partial waves for the scattering of 2.73 MeV neutrons from ^{12}C.

5. CONCLUSIONS

The MCAS approach to analyze (low-energy) nucleon-nucleus scattering is built from a model structure of the interaction potentials between a nucleon and each of the target states taken into consideration. All resonances (narrow and broad) in the cross section within the selected (positive) energy range can be found on a background. With negative energies, the MCAS method specifies the sub-threshold bound states of the compound nucleus. Inherent is a resonance finding procedure by which all resonances (and sub-threshold bound) states will be found within a chosen energy range. Also, when the OPP is used in definition of the Sturmians, those functions are assured to be orthogonal to any single nucleon bound state that is Pauli blocked. That allowance for the influence of the Pauli principle was crucial in finding results that concur well with observation.

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